Attribute and Variable

Sampling Plan Design

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These notes are posted at *www.mmbstatistical.com/Notes/VSPs.pdf*.

Agenda

1. Review sampling plan design for attribute inspections
2. Variables sampling plan design and operation
3. Comparison of sample sizes
4. What if we don't know σ?

Sampling Plan Goal

- The goal of any sampling plan is to distinguish good lots from bad lots.
- The observations may be attribute or variable.
- The formal hypotheses being tested are:

$$H_0 : p = p_0 \text{ (the lot is good)}$$

$$H_A : p > p_0 \text{ (the lot is bad)}$$

where p is the lot's true fraction defective.

Attribute Sampling Plan

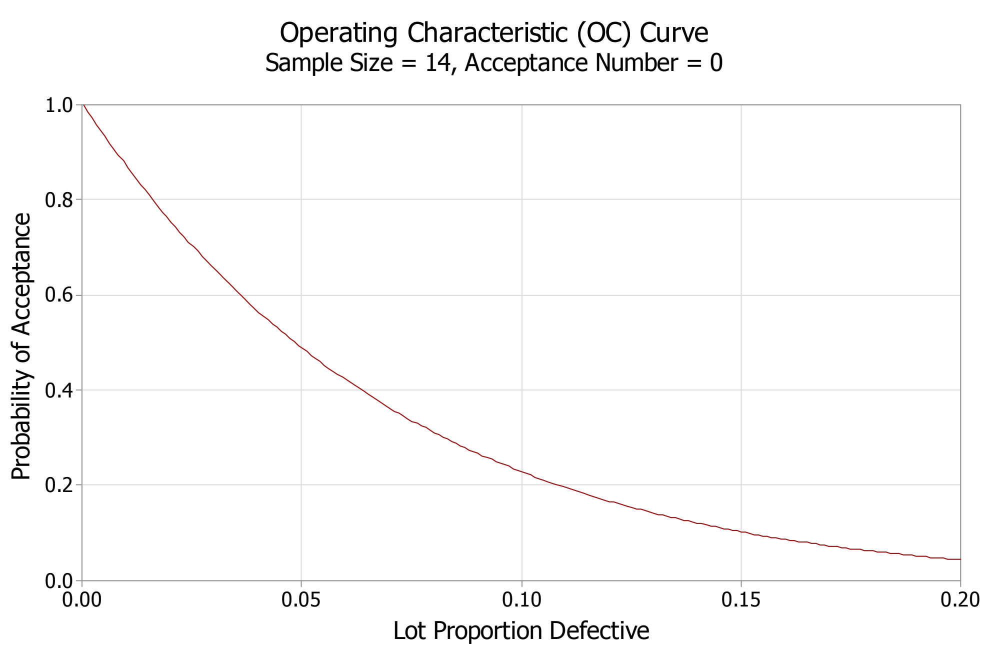
- In attribute sampling each unit inspected is judged to be good or bad.
- Attribute sampling plans are characterized by their sample size n and an acceptance number c.
- Attribute sampling plan operation:
  - Draw a random sample of size n from the lot.
  - Inspect and count the number of defective units D in the sample.
  - If $D > c$ then reject the lot. If $D \leq c$ then accept the lot.

Attribute Sampling Plan: Example

Example: What decision should be made if an attribute sampling plan with $n = 198$ and $c = 4$ finds the following number of defectives in random samples?

1. **a**. $D = 0$
   b. $D = 1$
   c. $D = 4$
   d. $D = 5$
   e. $D = 8$

Attribute Sampling Plan: OC Curve



Attribute Sampling Plan Design

We can design an attribute sampling plan by choosing two points on its OC curve:

- Acceptable Quality Level (AQL) condition: We want the plan to have a high probability of accepting lots with $p = AQL$.
- Rejectable Quality Level (RQL) condition: We want the plan to have a low probability of accepting lots with $p = RQL$.

Attribute Sampling Plan Design

- The AQL and RQL conditions provide two equations with two unknowns (n and c):

$$b(c; n, p = AQL) = 1 - \alpha$$

$$b(c; n, p = RQL) = \beta$$

  - $b(c; n, p)$ is the cumulative binomial distribution
  - α is the type 1 error rate (the probability of rejecting good lots)
  - β is the type 2 error rate (the probability of accepting bad lots).
- We want both error rates to be low but the cost of type 1 errors (internal failures) is usually different from type 2 errors (external failures) so their values should be chosen independently based on the cost consequences of each failure type.
- The simultaneous solution to the two equations gives unique values for n and c.

Attribute Sampling Plan Design: Example

Problem: What sample size and acceptance number are required to accept 95% of lots with 1% defective and 10% of lots with 4% defective?

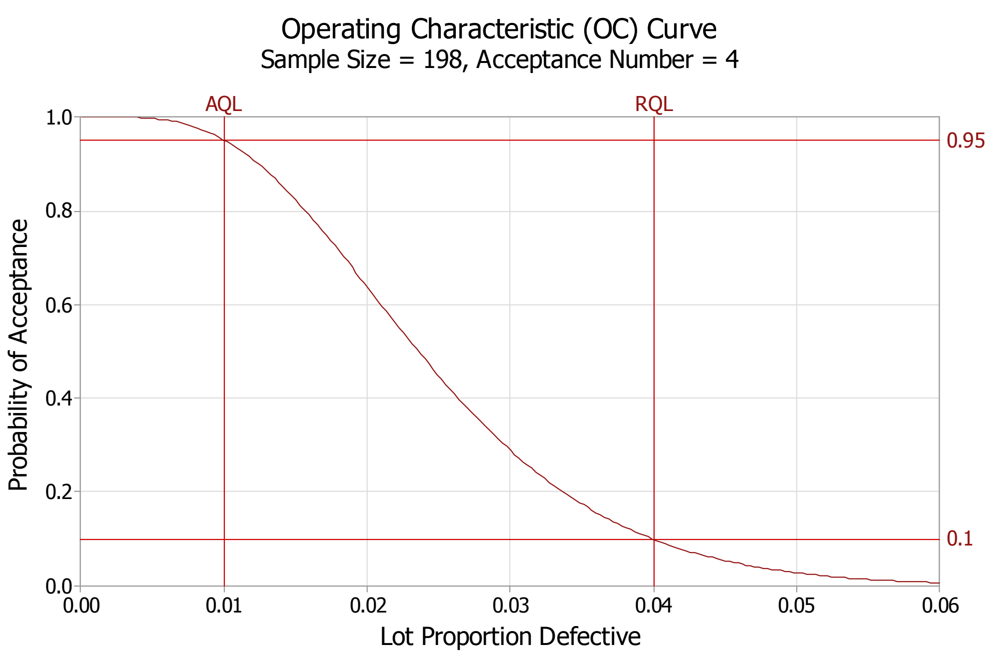
Solution: The simultaneous solution to:

$$b(c;n,p = AQL = 0.01) = 0.95$$

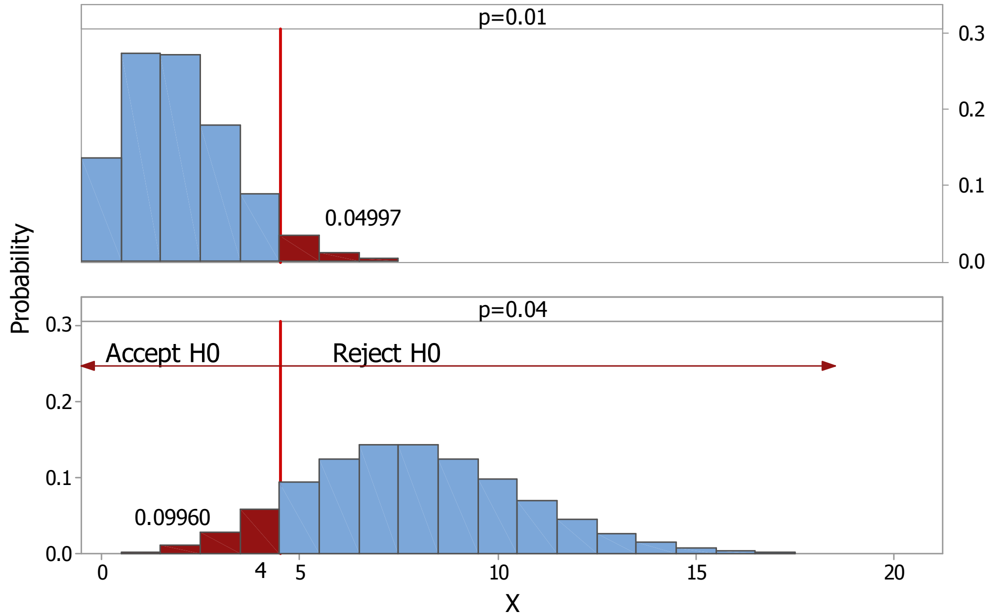
$$b(c;n,p = RQL = 0.04) = 0.10$$

can be determined by manual calculation (very painful), Larson's nomogram, or appropriate software.

Attribute Sampling Plan Design: Example

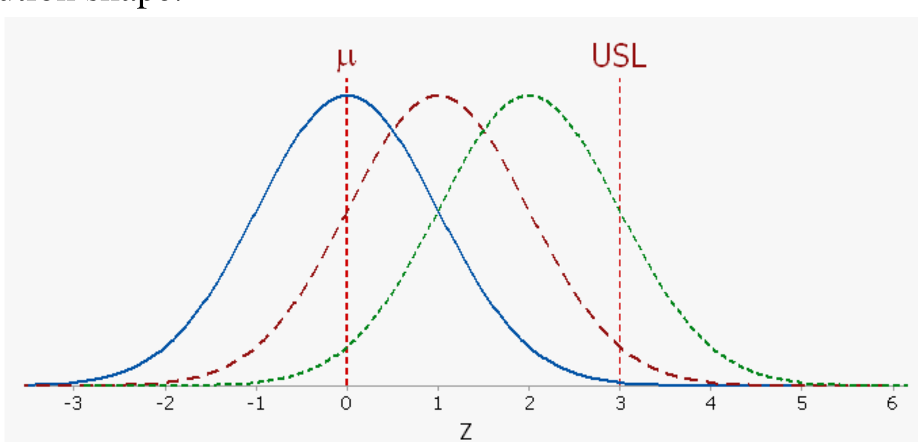


Attribute Sampling Plan Design: Example



Variables Sampling Plans

- Variables sampling plans (VSP) have the same goal as atttribute plans:
  - Accept lots with low fraction defective.
  - Reject lots with high fraction defective.
- Variables sampling plans use variables or measurement data instead of attribute data.
- The defective rate varies with the mean, standard deviation, and distribution shape.

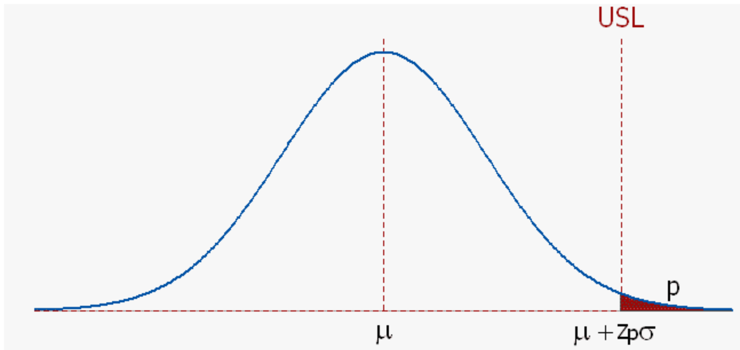


Variables Sampling Plans

When μ and σ are known and the distribution is normal the fraction defective p relative to the one-sided upper specification limit USL is

$$z_p = \frac{USL - \mu}{\sigma}$$

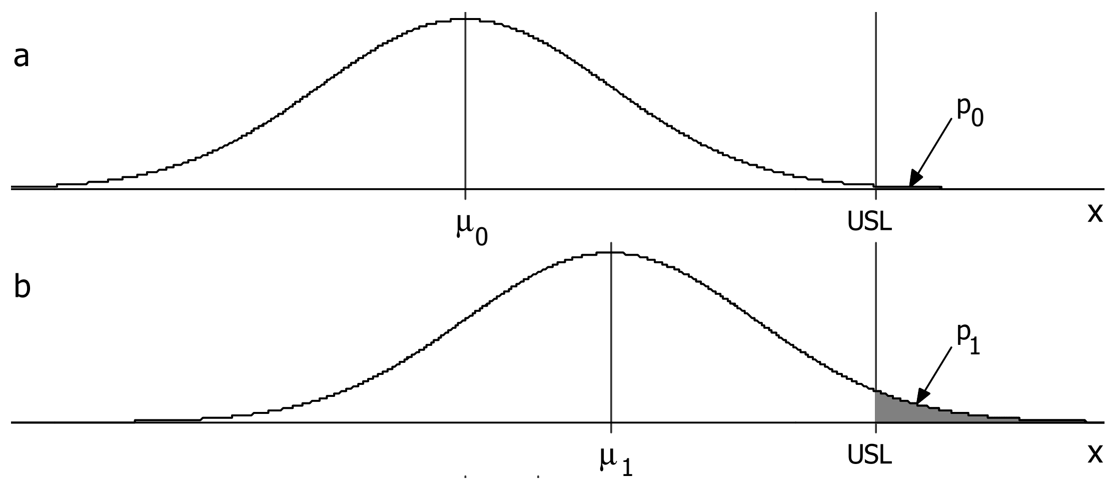
where p is the tail area under the normal curve.



The random sample in a VSP is used to estimate the population mean ($\bar{x}$ estimates μ) and maybe the standard deviation (s approximates σ).

Variables Sampling Plan: Design

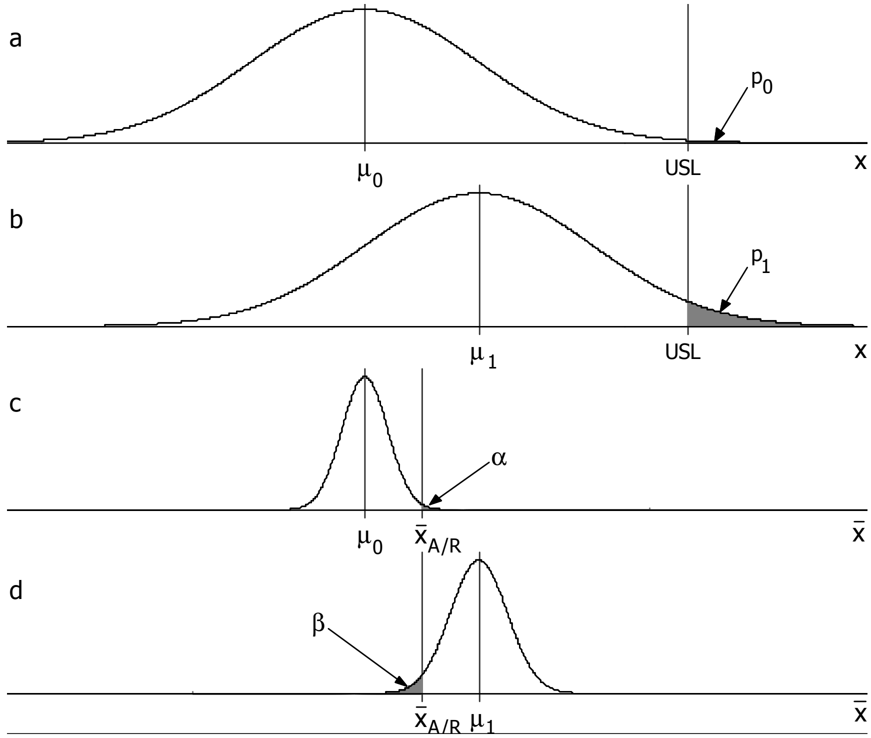
Suppose that we define AQL (p_0) and RQL (p_1) conditions:



If we know σ_x then at USL we can write

$$USL = \mu_0 + z_{p_0}\sigma_x = \mu_1 + z_{p_1}\sigma_x$$

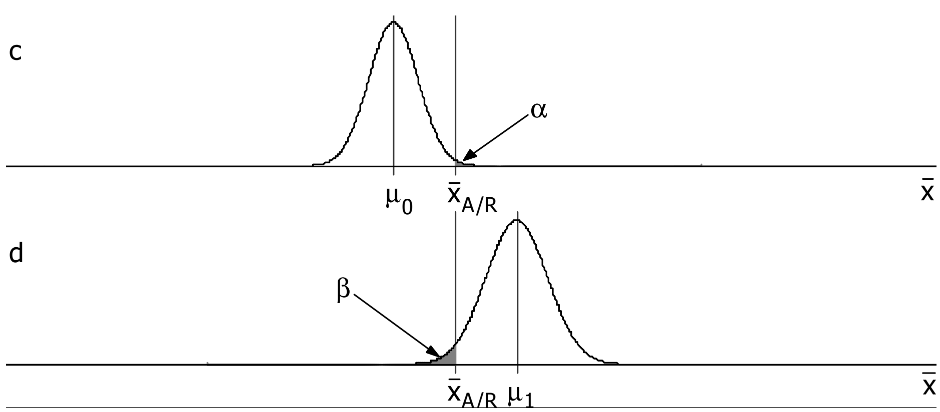
Variables Sampling Plan: Design



Variables Sampling Plan Design

From the distributions of sample means at $\bar{x}_{A/R}$ we can write:

$$\bar{x}_{A/R} = \mu_0 + z_\alpha \sigma_{\bar{x}} = \mu_1 - z_\beta \sigma_{\bar{x}}$$



Variables Sampling Plan: Design

If we solve the two equations:

$$USL = \mu_0 + z_{p_0}\sigma_x = \mu_1 + z_{p_1}\sigma_x$$

$$\bar{x}_{A/R} = \mu_0 + z_\alpha\sigma_{\bar{x}} = \mu_1 - z_\beta\sigma_{\bar{x}}$$

for the sample size n where $\sigma_{\bar{x}} = \sigma_x/\sqrt{n}$ we obtain:

$$n = \left(\frac{z_\alpha + z_\beta}{z_{p_0} - z_{p_1}}\right)^2$$

Variables Sampling Plan: Example

Problem: Find the variables sampling plan that will accept 95% of the lots with 1% defectives and reject 90% of the lots with 4% defectives when $\sigma = 30$ and the specification is one-sided with $USL = 700$.

Variables Sampling Plan: Example

Solution: The two specified points on the OC curve are $(p_0, 1-\alpha) = (0.01, 0.95)$ and $(p_1, \beta) = (0.04, 0.10)$. From the equation the required sample size is

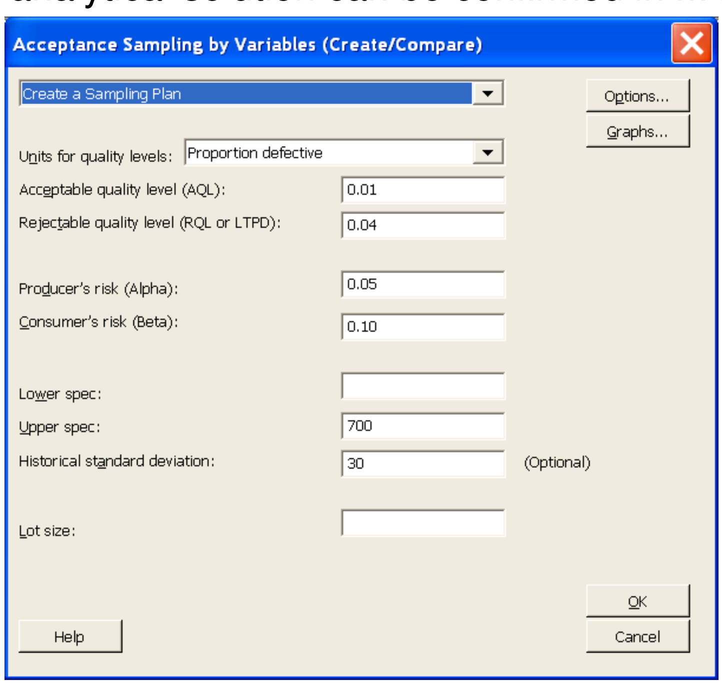
$$
\begin{aligned}
n &= \left(\frac{z_{0.05} + z_{0.10}}{z_{0.01} - z_{0.04}}\right)^2 \\
&= \left(\frac{1.645 + 1.282}{2.33 - 1.75}\right)^2 \\
&= 26
\end{aligned}
$$

and the critical value of $\bar{x}_{A/R}$ is

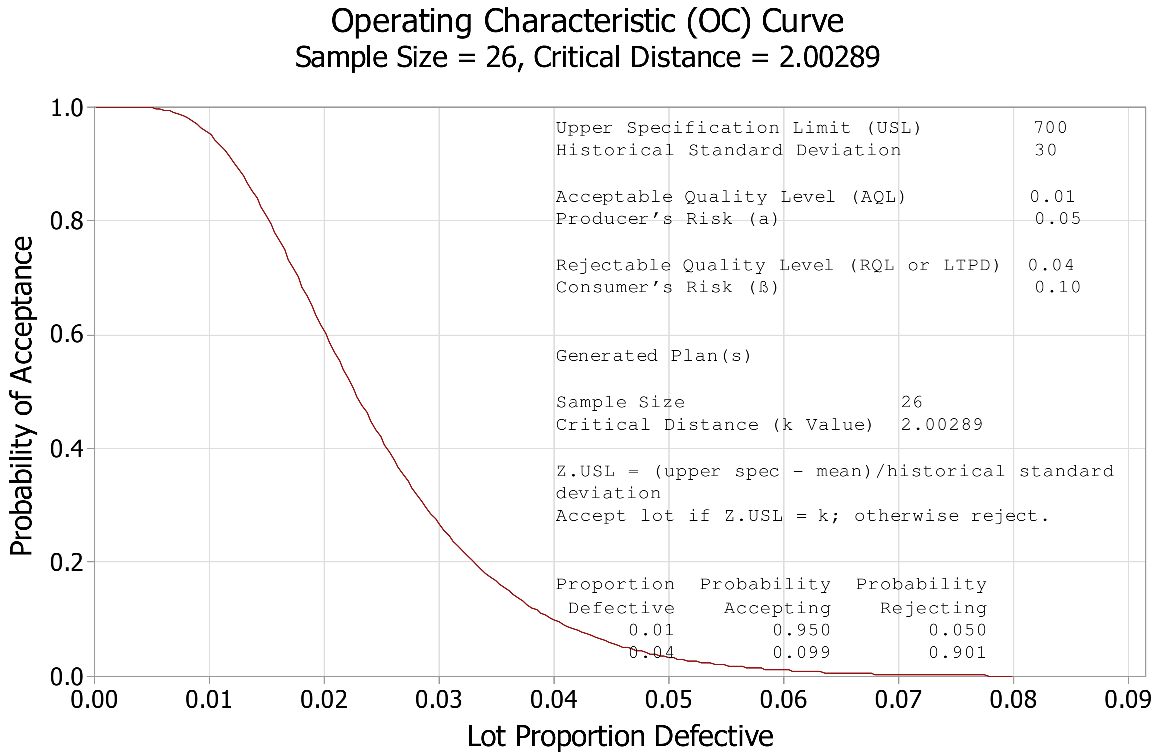
$$
\begin{aligned}
\bar{x}_{A/R} &= \mu_0 + z_\alpha \sigma_{\bar{x}} \\
&= (USL - z_{p_0}\sigma_x) + z_\alpha \frac{\sigma_x}{\sqrt{n}} \\
&= (700 - 2.33 \times 30) + 1.645 \frac{30}{\sqrt{26}} \\
&= 640
\end{aligned}
$$

Variables Sampling Plan: Example

Solution: The analytical solution can be confirmed in MINITAB:



Variables Sampling Plan: Example



Comparison of ASP to VSP Sample Sizes

Attribute and variables sampling plans can both be designed to meet the same AQL and RQL conditions. In that case the ratio of the sample sizes is given by

$$\frac{n_{attributes}}{n_{variables}} = \frac{\left(\frac{z_\alpha\sqrt{p_0(1-p_0)}+z_\beta\sqrt{p_1(1-p_1)}}{p_1-p_0}\right)^2}{\left(\frac{z_\alpha+z_\beta}{z_{p_0}-z_{p_1}}\right)^2}$$

For the special case of $\alpha = \beta$ and when p_0 and p_1 are both small, say, less than about 10%, this ratio simplifies and approximates to

$$\frac{n_{attributes}}{n_{variables}} \simeq \frac{1}{4}\left(\frac{z_{p_0} - z_{p_1}}{\sqrt{p_1} - \sqrt{p_0}}\right)^2$$

Comparison of ASP to VSP Sample Sizes

Example: Determine the sample size ratio for attributes and variables inspection plans that will accept 95% of the lots with 0.1% defectives and reject 95% of the lots with 0.4% defectives.

Solution: The two points on the OC curve are $(p_0 = 0.001, 1 - \alpha = 0.95)$ and $(p_1 = 0.004, \beta = 0.05)$. Because $\alpha = \beta = 0.05$ and both p_0 and p_1 are relatively small the ratio of the attributes- to variables-based sample sizes is approximately

$$\begin{aligned} \frac{n_{attributes}}{n_{variables}} &\simeq \frac{1}{4}\left(\frac{z_{0.001} - z_{0.004}}{\sqrt{0.004} - \sqrt{0.001}}\right)^2 \\ &\simeq \frac{1}{4}\left(\frac{3.090 - 2.652}{\sqrt{0.004} - \sqrt{0.001}}\right)^2 \\ &\simeq 48 \end{aligned}$$

Variables Sampling Plan: Design

When σ_x is unknown and must be estimated with s from the sample data the operating characteristic curve (p, P_A) for the sampling plan is characterized by the noncentral t distribution

$$t_{P_A, df, \phi} = -k\sqrt{n}$$

where P_A is the probability of accepting H_0, $df = n - 1$ is the degrees of freedom, and

$$\phi = -z_p\sqrt{n}$$

is the t distribution noncentrality parameter. For two specified points on the OC curve, $(p, P_A) = (p_0, 1 - \alpha)$ and $(p, P_A) = (p_1, \beta)$, the same values of k and n apply, so the unique sample size must satisfy the nightmarish condition

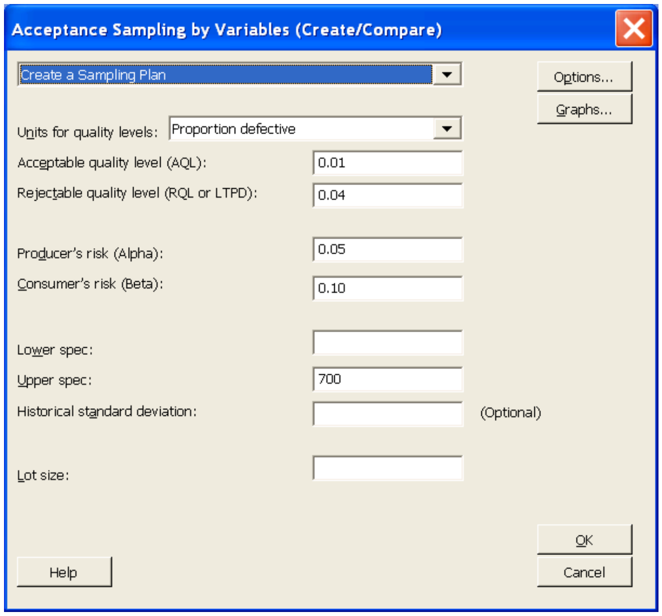
$$t_{1-\alpha, n-1, -z_{p_0}\sqrt{n}} = t_{\beta, n-1, -z_{p_1}\sqrt{n}}$$

This problem is best left to software.

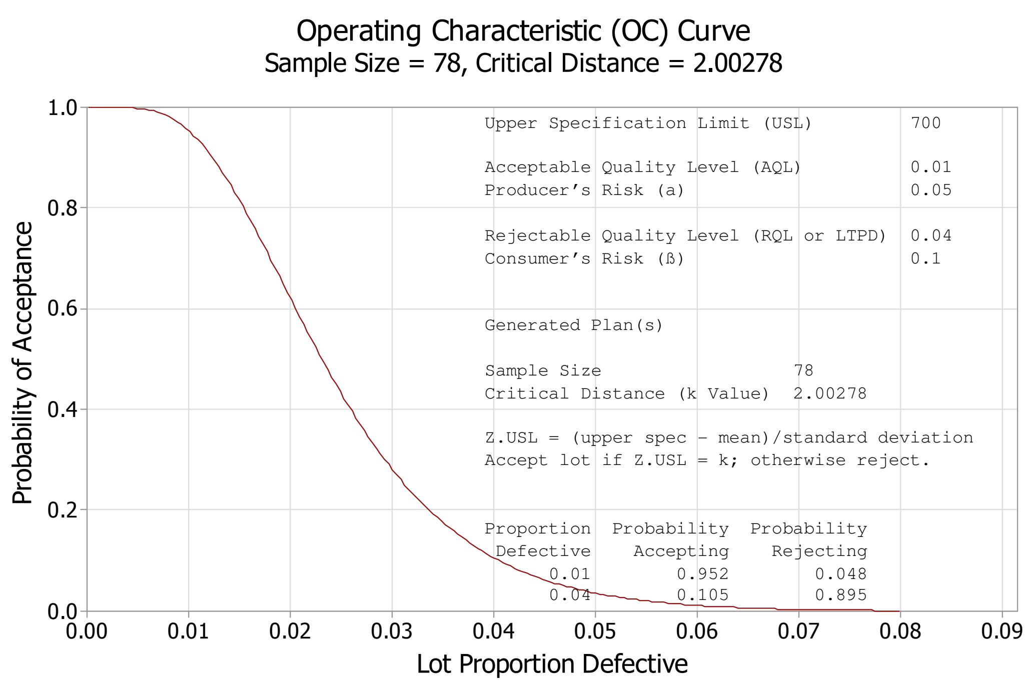
Variables Sampling Plan: Example

Problem: Recalculate the sample size for the variables sampling plan under the condition that the standard deviation is unknown.

Solution:



Variables Sampling Plan: Example



References

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2. Montgomery, Introduction to Statistical Quality Control
3. Grant and Leavenworth, Statistical Quality Control
4. ANSI/ASQ Z1.4 Sampling Procedures and Tables for Inspection by Attributes
5. ANSI ASQ Z1.9 Sampling Procedures and Tables for Inspection by Variables for Percent Defective