Rectifying Inspection and Quality Cost

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Acceptance Sampling

- *Acceptance sampling* uses sampling inspection to determine whether to accept or reject lots.
  - If the quality of the sample is good then we accept the lot.
  - If the quality of the sample is bad then we reject the lot.
- Rejected lots may be:
  - scrapped
  - sent back to the supplier
  - 100% inspected (*rectifying inspection*).

Sampling Plan Design

- Sampling plan design is based on statistical hypothesis testing methods: H_0: *the lot is good* versus H_A: *the lot is bad*. Sample data are used to determine which hypothesis to accept.
- Sampling plan parameters (the sample size and acceptance criterion) are chosen to manage risks or error rates. There are two:
  - Type 1 error, false alarm, manufacturer's risk - rejecting a good lot - indicated with the symbol α
  - Type 2 error, missed alarm, consumer's risk - accepting a bad lot - indicated with the symbol β

Sampling Plan Design

Sampling can be done using attribute or variable data:

- Example (attributes plan): A sampling plan for defectives uses a sample size of $n = 40$ and has an acceptance number of $c = 1$ (i.e. accept the lot if there are zero or one defectives in the sample, reject the lot if there are two or more defectives in the sample).
- Example (variables plan, single-sided spec): A sampling plan for a variable/measurement response uses as sample size of $n = 12$ and has acceptance criterion $k = 2.2$ (i.e. accept the lot if $(USL - \bar{x})/s > k$ or $(\bar{x} - LSL)/s > k$).

Attribute Sampling Plan Design

- For an attribute sampling plan for defectives the probability of accepting (P_A) lots with fraction defective p is given by the cumulative binomial probability

$$P_A = \sum_{x=0}^{c} b(x;n,p)$$
$$= b(c;n,p)$$

- Attribute sampling plans for defectives are designed by choosing two (p, P_A) pairs:
  - The low fraction defective, called the *acceptable quality level* or $p = AQL$, that the process can tolerate and the corresponding high probability of accepting lots of AQL quality $P_A = (1-\alpha)$.
  - The high fraction defective, called the *rejectable quality level* or $p = RQL$, that the process cannot tolerate and the corresponding low probability of rejecting lots of RQL quality $P_A = \beta$.

Attribute Sampling Plan Design

- Putting this information together gives two equations:
  - For ($p = AQL, P_A = 1-\alpha$):

$$1-\alpha = b(c;n,p = AQL)$$

  - For ($p = RQL, P_A = \beta$):

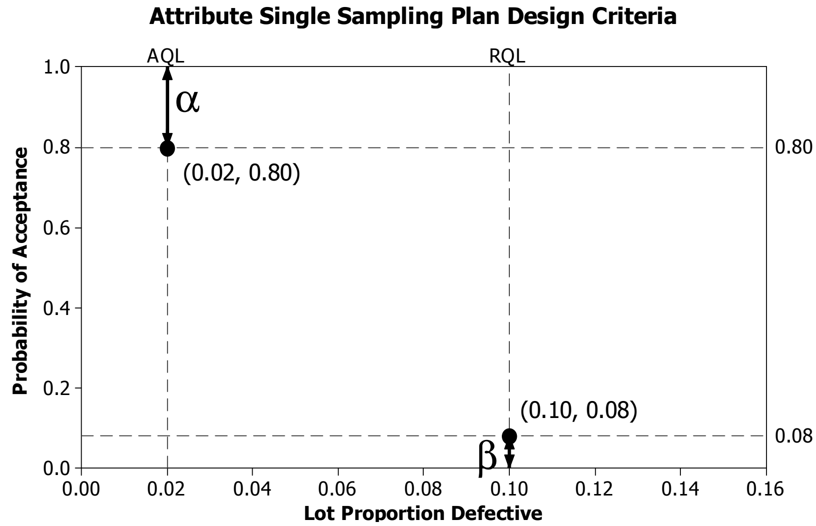
$$\beta = b(c;n,p = RQL)$$

- The simultaneous solution to the two equations gives values for the two unknowns - the sampling plan's sample size n and the acceptance number c.
- Solving these equations is done graphically using Larson's nomogram (an archaic method but still in use) or special software, e.g. MINITAB **Stat**> **Quality Tools**> **Acceptance Sampling by Attributes**> **Create a Sampling Plan**.

Attribute Sampling Plan Design

Example: Determine the sampling plan parameters n and c for the attribute sampling plan that must accept 80% of lots with 2% defectives and accept 8% of lots with 10% defectives.

Solution:



Attribute Sampling Plan Design

Using MINITAB:

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Acceptable Quality Level (AQL)             0.02
Producer's Risk (alpha)                    0.2

Rejectable Quality Level (RQL or LTPD)     0.1
Consumer's Risk (beta)                     0.08

Generated Plan(s)
Sample Size         41
Acceptance Number    1

Proportion  Probability  Probability
 Defective    Accepting    Rejecting
      0.02        0.802        0.198
      0.10        0.074        0.926
```

Operating Characteristic Curve

- The *operating characteristic* (OC) curve of a sampling plan with specified n and c is the plot of P_A versus p where

$$P_A = b(c; n, p)$$

- OC curves are useful to
  - Study the performance of the sampling plan
  - Compare the performanc of two or more sampling plans

Operating Characteristic Curve

Example: Determine the operating characteristic (OC) curve for the $n = 40$, $c = 1$ attribute sampling plan.

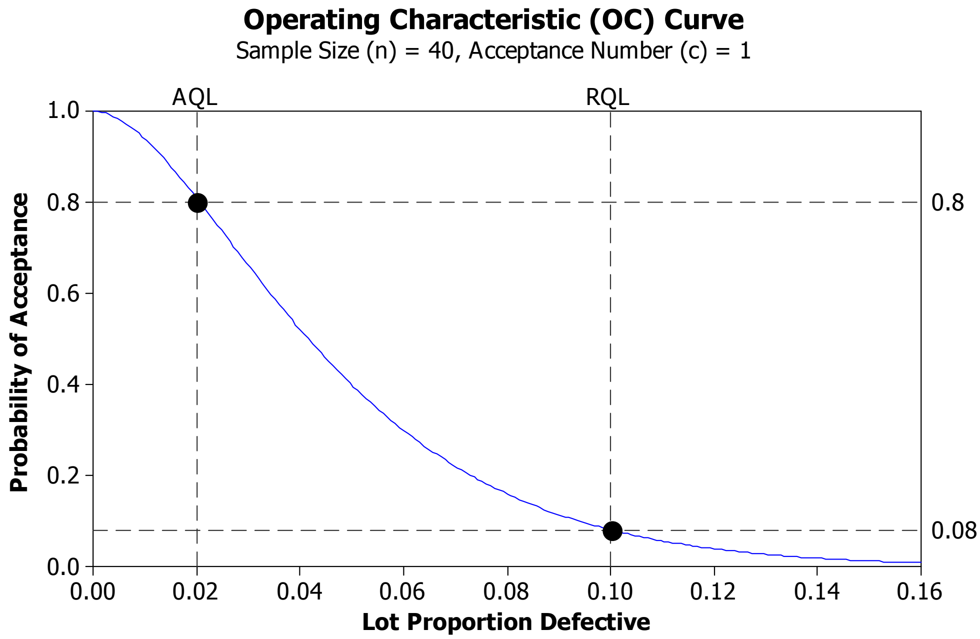
Solution: For the $n = 40$, $c = 1$ sampling plan

$$P_A = b(c = 1; n = 40, p).$$

Some values of P_A as a function of p are

p	0.000	0.01	0.02	0.04	0.08	0.10
P_A	1.000	0.939	0.802	0.521	0.159	0.074

Operating Characteristic Curve



Average Total Inspection

- The number of units to be inspected will be either:
  - n if the lot is accepted
  - N if the lot is rejected
- For a series of lots, the average number of units inspected will depend on the fraction defective and will generally fall between n and N. This average, called the *average total inspection* (ATI) or *average sample number* (ASN), is given by

$$ATI = nP_A + N(1 - P_A)$$

- ATI is useful for managing the inspection process.

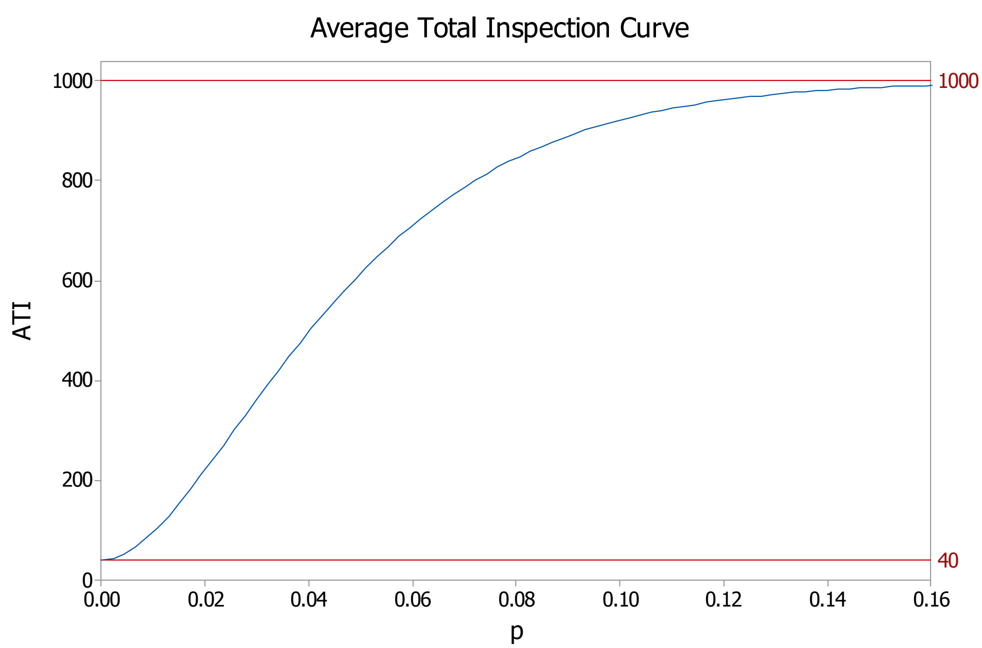
Average Total Inspection

Example: Plot ATI versus the fraction defective for the $n = 40$, $c = 1$ plan using a lot size of $N = 1000$.

Solution: From the equation for ATI, some values of ATI versus p are:

p	0.000	0.01	0.02	0.04	0.08	0.16
P_A	1.000	0.939	0.810	0.521	0.159	0.008
ATI	40	98	222	500	847	992

Average Total Inspection



Average Outgoing Quality

- p is the fraction defective of the material coming into the inspection operation.
- The post-inspection fraction defective or *average outgoing quality* (AOQ) is given by

$$AOQ = pP_A\left(\frac{N-n}{N}\right)$$

- When p is small, near 0, the sampling plan will accept most lots and then $AOQ \simeq p$.
- When p is large, so that the sampling plan rejects most lots, then those lots will be 100% inspected and $AOQ \simeq 0$.
- AOQ reaches a maximum value when p is moderate. The maximum AOQ is called the *average outgoing quality limit* ($AOQL$). Some sampling standards, e.g. Dodge-Romig, are indexed by $AOQL$ values.

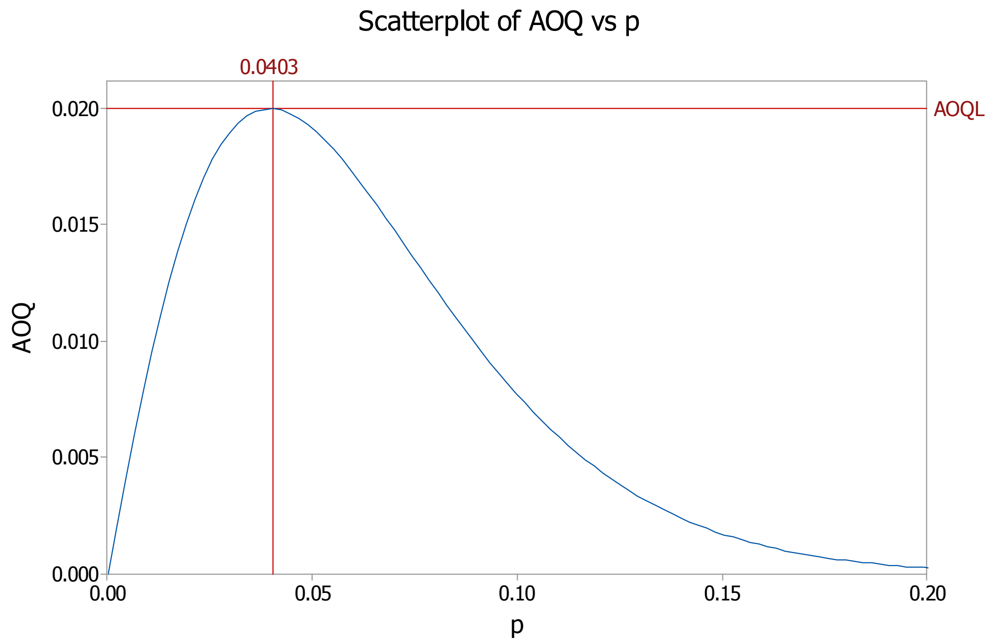
Average Outgoing Quality

Example: Plot the AOQ curve for the $n = 40$, $c = 1$ plan using a lot size of $N = 1000$. Identify the $AOQL$ value and its corresponding process fraction defective.

Solution: From the equation for AOQ, some values of AOQ versus p are:

p	0.000	0.01	0.02	0.04	0.08	0.16
P_A	1.000	0.939	0.810	0.521	0.159	0.008
AOQ	0.000	0.0090	0.0155	0.0204	0.0122	0.0012

Average Outgoing Quality



Inspection Yield

- The fraction of the units that pass the rectifying inspection process is the *inspection yield* (Y) given by

$$Y = P_A + (1 - P_A)(1 - p)$$

- When the fraction defective p is small, near 0, then the sampling plan will pass most lots and $Y \simeq 1$.
- When the fraction defective p is large then the sampling plan will reject most lots. Only the good units will pass the rectifying inspection operation so $Y \simeq 1 - p$.

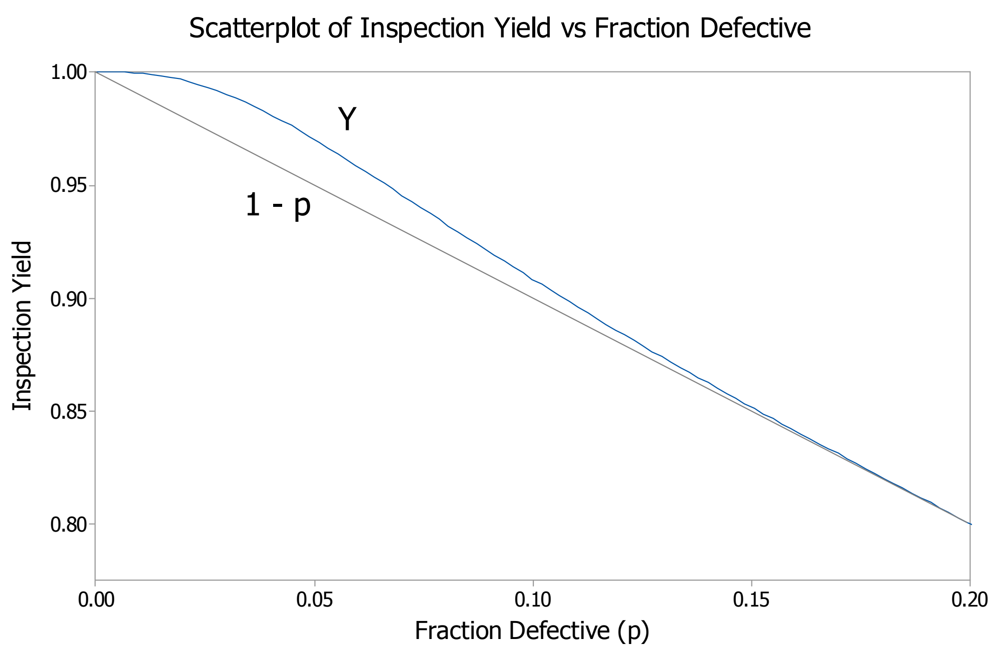
Inspection Yield

Example: Plot the yield curve for the $n = 40$, $c = 1$ plan.

Solution: From the equation for inspection yield, some values of Y versus p are:

p	0.000	0.01	0.02	0.04	0.08	0.16
P_A	1.000	0.939	0.810	0.521	0.159	0.008
Y	1	0.999	0.996	0.981	0.932	0.841

Inspection Yield



Net Income

- The OC curve interpretation of the sampling plan's performance is incomplete. We must look at it in terms of quality cost.
- The costs that must be considered are (cost per unit):
  - Material and Labor Cost (M)
  - Inspection Cost (I)
  - Selling Price (S)
  - External Failure Cost (F)
- Net income is calculated using the expection value method:

$$Net\ Income = \sum_{\text{all } i} Cost_i \times P_i$$

where $Cost_i$ is the cost associated with cost item i for the entire lot and P_i is the probability of incurring cost item i.

Net Income

For the rectifying inspection process:

i	$Cost_i \times P_i$
Material and Labor	$-M \times N$
Inspection	$-I \times ASN$
Sales	$S \times Y \times N$
External Failure	$-F \times N \times AOQ$
Net Income	$\sum_i Cost_i \times P_i$

Net Income

Example: Calculate the net income for the $n = 40$, $c = 1$ plan using a lot size of $N = 1000$ when $p = 0.04$. The per unit material and labor cost is $M = \$5$, the inspection cost is $I = \$1$, the sales price is $S = \$20$, and the external failure cost is $F = \$6$ (replace all bad units with good ones, no shipping charges).

Solution: From the equation for net income:

i	$Cost_i \times P_i$	$Cost_i \times P_i$	$Cost_i \times P_i$
M & L	$-M \times N$	$-\$5 \times 1000$	$-\$5000$
Inspection	$-I \times ASN$	$-\$1 \times 500$	$-\$500$
Sales	$S \times Y \times N$	$\$20 \times 0.981 \times 1000$	$\$19620$
Ext. Failure	$-F \times N \times AOQ$	$-\$6 \times 1000 \times 0.0204$	$-\$122$
Net Income	$\sum_i Cost_i \times P_i$		$\$13998$

Reference Inspection Processes

Every sampling plan should be compared to two reference inspection processes:

- 100% inspection
- No inspection

Reference Inspection Processes - 100% Inspection

i	$Cost_i \times P_i$	$Cost_i \times P_i$	$Cost_i \times P_i$
M & L	$-M \times N$	$-\$5 \times 1000$	$-\$5000$
Inspection	$-I \times ASN$	$-\$1 \times 1000$	$-\$1000$
Sales	$S \times (1 - p) \times N$	$\$20 \times (1 - 0.04) \times 1000$	$\$19200$
Ext. Failure	$-F \times N \times AOQ$	$-\$6 \times 1000 \times 0$	$-\$0$
Net Income	$\sum_i Cost_i \times P_i$		$\$13200$

Reference Inspection Processes - No Inspection

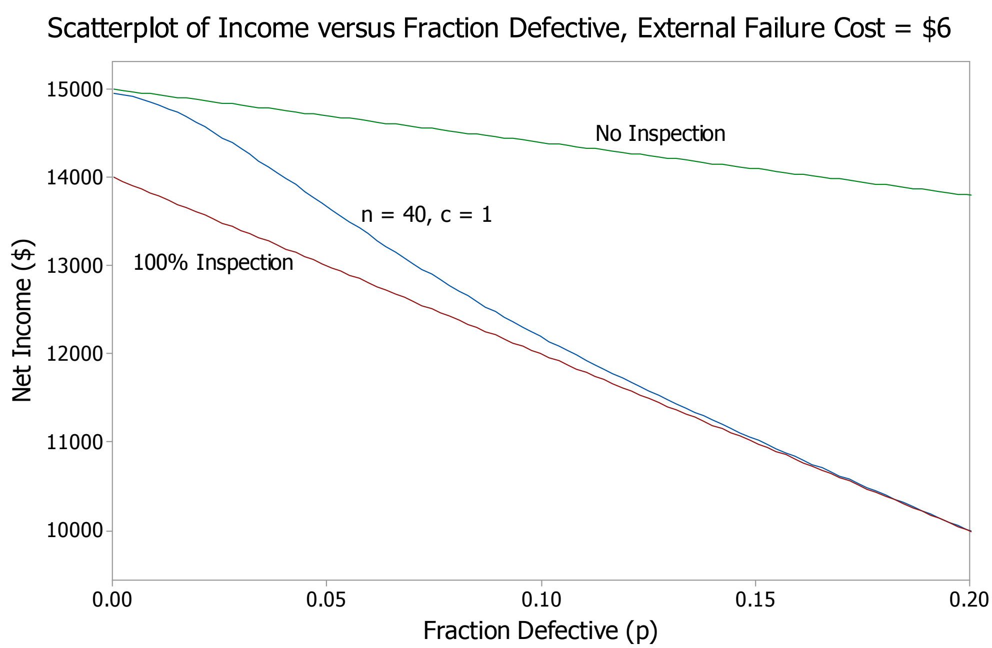
i	$Cost_i \times P_i$	$Cost_i \times P_i$	$Cost_i \times P_i$
Material and Labor	$-M \times N$	$-\$5 \times 1000$	$-\$5000$
Inspection	$-I \times ASN$	$-\$1 \times 0$	$-\$0$
Sales	$S \times Y \times N$	$\$20 \times 1 \times 1000$	$\$20000$
Ext. Failure	$-F \times N \times AOQ$	$-\$6 \times 1000 \times 0.04$	$-\$240$
Net Income	$\sum_i Cost_i \times P_i$		$\$14760$

Back to Net Income

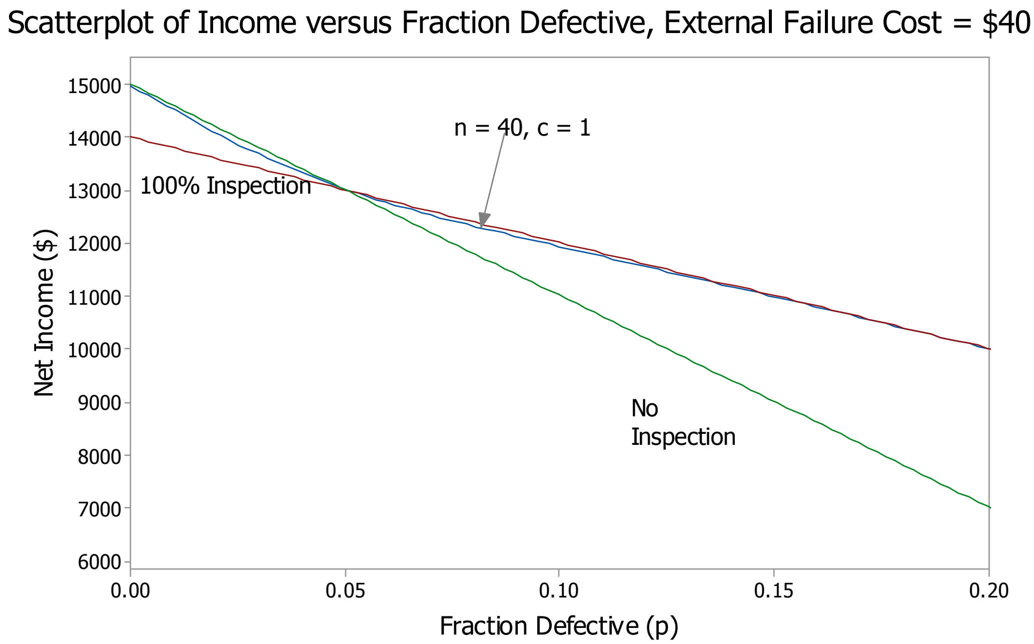
We always want to maximize our net income. Which sampling strategy does that when $p = 0.04$? What if you don't know p?

i	$n = 40, c = 1$	100% Insp.	No Insp.
M & L	–\$5000	–\$5000	–\$5000
Inspection	–\$500	–\$1000	–\$0
Sales	\$19620	\$19200	\$20000
Ext. Failure	–\$122	–\$0	–\$240
Net Income	\$13998	\$13200	\$14760

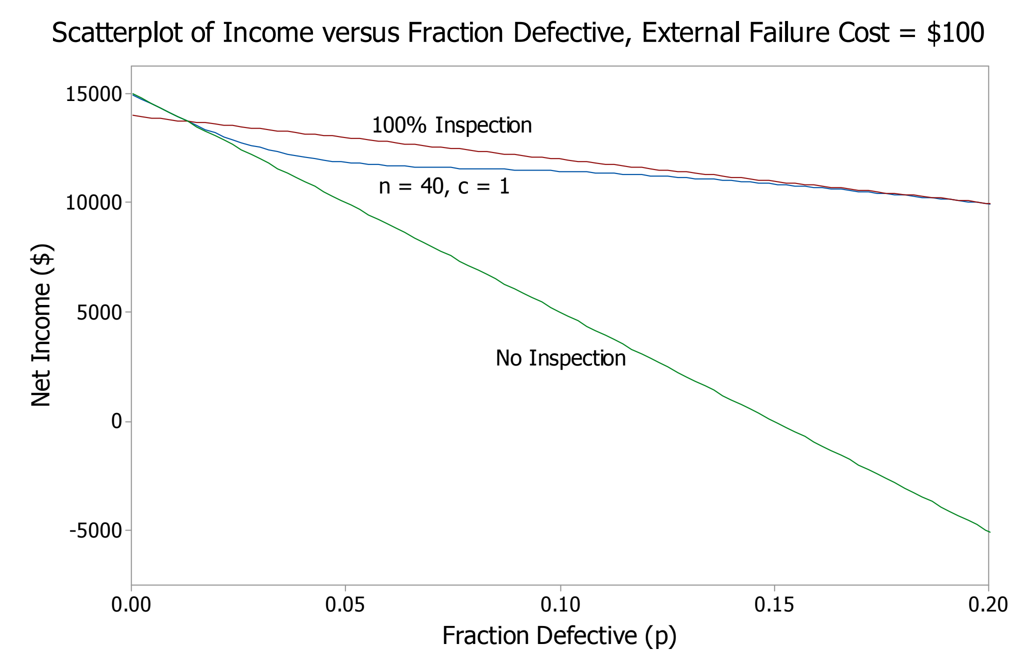
Net Income with \$6 Ext. Failure Cost



Net Income with $40 Ext. Failure Cost



Net Income with $100 Ext. Failure Cost



Conclusions

- The optimal sampling method is either no inspection or 100% inspection. The problem is determining which method is correct at the moment.
- The factors that determine the optimal sampling method (the one that maximizes net income) are the external failure cost and the process's fraction defective.
- If the external failure cost is very low (e.g. replace failed units with good ones) then don't inspect at all - just ship the product.
- If the external failure cost is moderate (e.g. refund the sale price of the failed unit and replace it with a good one) or very high then use no inspection if the fraction defective is very low or 100% inspection if the fraction defective is very high. When the fraction defective is unknown the sampling plan approximates the best features of the no inspection and 100% inspection sampling plans.

References

- Papadakis, Journal of Quality Technology, July 1985, Vol. 17, No. 3, p. 121-127, The Deming Inspection Criterion for Choosing Zero or 100 Percent Inspection (http://asq.org/data/subscriptions/jqt_open/1985/july/jqtv17i3papadakis