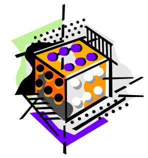
Expectation Value

If a game of chance pays amounts a_1, a_2, ..., a_n and the probabilities of winning these amounts are p_1, p_2, ..., p_n then in a long series of plays the expected earnings per play are given by:

$$E = \sum_{i=1}^{n} a_i p_i$$

Expectation Value 

Example: In a game of chance players roll a single die. If they roll an odd number they get that many dollars (e.g. rolling a 3 pays $3). If they roll an even number they lose. What are the expected earnings? Should you play to win if it costs $1 to play? $2?

Solution: The expected earnings are:

$$
\begin{aligned}
E &= \sum_{i=1}^{n} a_i p_i \\
&= \$1(\tfrac{1}{6}) + \$0(\tfrac{1}{6}) + \$3(\tfrac{1}{6}) + \$0(\tfrac{1}{6}) + \$5(\tfrac{1}{6}) + \$0(\tfrac{1}{6}) \\
&= \tfrac{1}{6} + 0 + \tfrac{3}{6} + 0 + \tfrac{5}{6} + 0 \\
&= \$1.50
\end{aligned}
$$

At $1 the game would be profitable, but at $2 you're going to lose!

Expectation Value

Example: A card game is played by drawing a single card from a standard deck. The game pays \$1 if you draw a face card and \$5 if you draw an ace. Should you play to win if it costs \$1 to play?

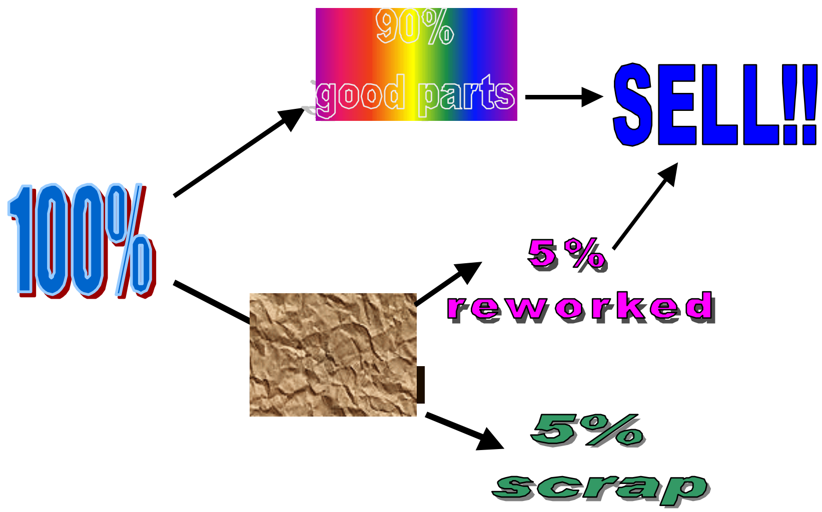
$$
\begin{aligned}
E &= \sum_{i=1}^{n} a_i p_i \\
&= (-\$1)(1) + \$1\left(\tfrac{12}{52}\right) + \$5\left(\tfrac{4}{52}\right) \\
&= (-1) + \tfrac{12}{52} + \tfrac{20}{52} \\
&= -\$0.385
\end{aligned}
$$

On average you will lose $\$0.39$ each time you play.

Expectation Value

Example: A manufacturing business produces parts that use $20 in material and labor. 90% of the parts produced are good and can be sold for full price at $80 each. Of the 10% of the parts that are bad, half can be reworked at a cost of $8 each and sold for full price. The remaining 5% must be scrapped. Scrapped parts can be sold for salvage at $2 each. Find the expected earnings per part and determine the cost of poor quality (COPQ).

Solution:



$$E = \sum_{i=1}^{n} a_i p_i$$

$$= material + rework + salvage + sales$$

$$= (-\$20 \times 1.0) + (-\$8 \times 0.05) + (+\$2 \times 0.05) + (+\$80 \times 0.95)$$

$$= (-20.00) + (-0.40) + (0.10) + (76.00)$$

$$= \$55.70$$

Expected earnings are $\$55.70$ per part manufactured. If there were no quality problems, then the expected earnings per part would be:

$$E = 1.0 \times (-\$20) + 1.0 \times \$80 = \$60.00$$

The COPQ is the difference between the two expected earnings:

$$COPQ = \$60.00 - \$55.70 = \$4.30$$

Problem: Determine if the rework operation is cost effective assuming that parts that would have been reworked will just be sold for scrap instead.

Solution: The new model is:

$$\begin{aligned} E &= \sum_{i=1}^{n} a_i p_i \\ &= material + salvage + sales \\ &= (-\$20 \times 1.0) + (+\$2 \times 0.10) + (+\$80 \times 0.90) \\ &= (-20.00) + (2) + (72.00) \\ &= \$54.00 \end{aligned}$$

so the cost of poor quality without the rework operation is:

$$COPQ = \$60.00 - \$54.00 = \$6.00$$

The rework operation is cost effective because it saves about $\$1.70$ per part started.

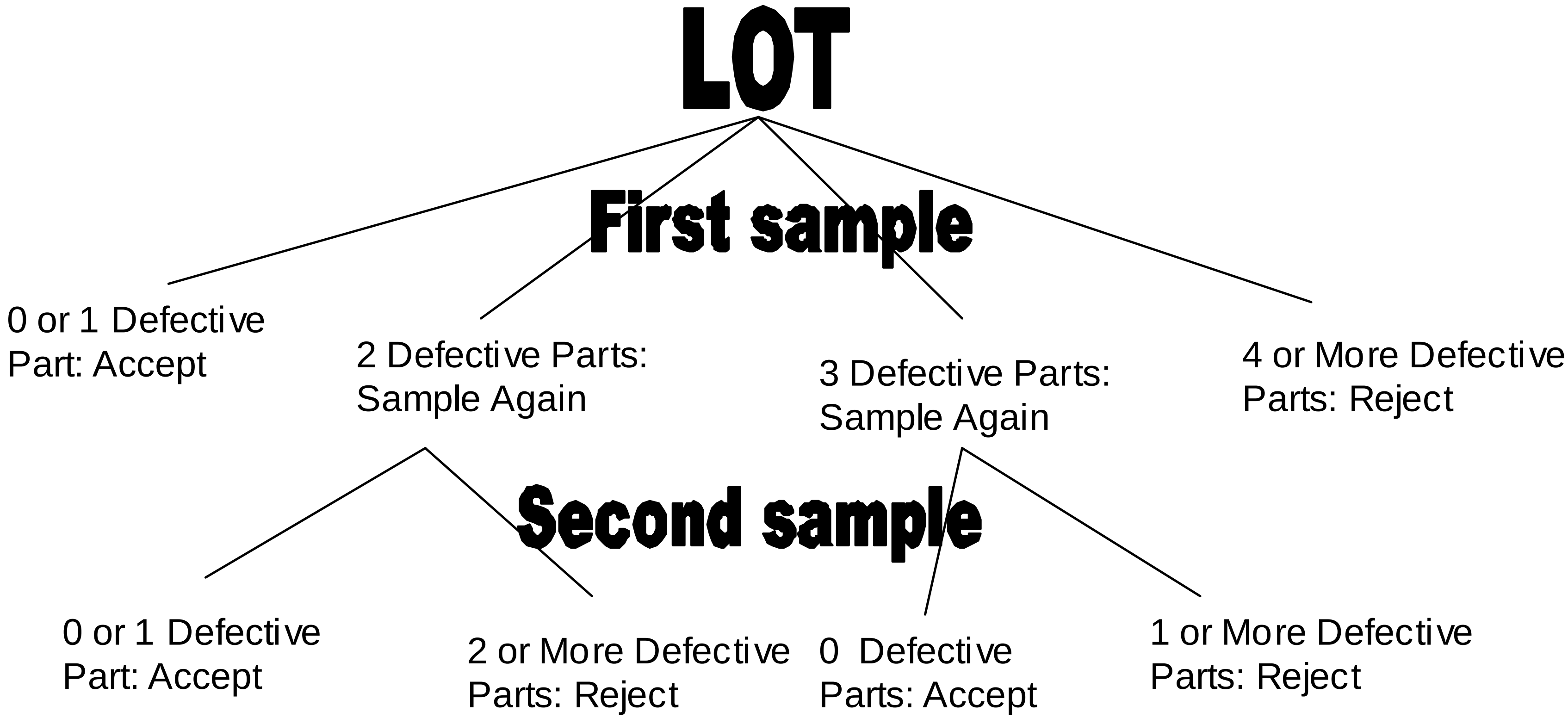
Expectation Value

Example: A double sampling plan for defectives has sampling parameters $n_1 = 80$, $n_2 = 160$, $c_1 = 1$, and $c_2 = 3$. For incoming lot fraction defective $p = 0.01$ the probability of:

1. Accepting the lot on the first sample is $P_A^I = 0.809$.
2. Rejecting the lot on the first sample is $P_R^I = 0.009$.
3. Accepting the lot on the second sample is $P_A^{II} = 0.083$.
4. Rejecting the lot on the second sample is $P_R^{II} = 0.099$.

Find the average sample number (ASN) for this condition. (The ASN is the expected number of samples taken under these conditions. Sampling is always complete.)

* The first sample can not be used in the second sample.


LOT
First sample
0 or 1 Defective Part: Accept
2 Defective Parts: Sample Again
3 Defective Parts: Sample Again
4 or More Defective Parts: Reject
Second sample
0 or 1 Defective Part: Accept
2 or More Defective Parts: Reject
0 Defective Parts: Accept
1 or More Defective Parts: Reject

Solution: Let $P^I = P^I_A + P^I_R = 0.818$ be the probability of making a decision on the first sample and $P^{II} = P^{II}_A + P^{II}_R = 0.182$ be the probability of making a decision on the second sample. Then:

$$\begin{aligned} ASN &= n_1(1) + n_2(P^{II}) \\ &= 80(1) + 160(0.182) \\ &= 109.1 \end{aligned}$$

Alternatively:

$$\begin{aligned} ASN &= n_1(P^I) + (n_1 + n_2)(P^{II}) \\ &= 80(0.818) + (80 + 160)(0.182) \\ &= 109.1 \end{aligned}$$