1 Design of Experiments

Part 1: A Trebuchet Experiment (4 February 2005)
Part 2: Documenting and Reporting the Experiment (25 February 2005)

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1.1 Experiment Documentation

There are three distinct levels of documentation possible in any DOE program:

- Comprehensive collection of documents created during the DOE program process
- Presentation, e.g. to Management
- Formal report

1.2 General Procedure for Experimentation

1. Cause and effect analysis
2. Process flow mapping
3. Problem statement
4. Preliminary experiments
5. Experiment design selection
6. Sample size, blocking, and randomization
7. Experiment execution

8. Analysis
9. Interpretation
10. Confirmation experiment
11. Reporting the results

1.3 General Procedure for Experimentation

1. Cause and Effect Analysis

- Catalog all of the input variables: methods, manpower, machines, material, and environment
- Catalog all of the responses
- Make the catalogs exhaustive!
- Variables may be quantitative or qualitative
- Brainstorm everything
- The goal is to form a **complete** list of **all** input and output variables
- Best formats for presentations are probably:
  - Input/Process/Output IPO diagram
  - Cause and effect matrix

1.3.1 Trebuchet DOE: IPO

Process: trebuchet launch
Inputs:

- Machine: L_1, L_2, L_3, W_1, W_2, θ, η, friction, L_{W_1}
- Methods: basic trebuchet setup, launch configuration, launch execution, response measurement
- Materials: trebuchet tower, trebuchet arm, counterweight, counterweight bridle, projectile, projectile bridle, projectile guide, table, measurement device
- Manpower: administrator, configuration, operation, measurement, records
- Environment: room dimensions (3), floor material, obstructions

1.3.2 Trebuchet DOE: IPO

Outputs

- Horizontal range (±):
  - Point of first impact
  - Bounces
  - Slides
  - Rolls
- Altitude
- Efficiency
- Interference
- Other special causes

1.4 General Procedure for Experimentation

2. Document the Process to be Studied

- Flow charts
- Written procedures
- Identify workmanship examples
- Identify training opportunities
- Talk to the operators or technicians who do the work
- Get general agreement on all steps of the process
- Determine level of detail: documentation should be written for someone "skilled in the art", i.e. trained.

1.4.1 Trebuchet DOE: Process

1. Set up the trebuchet base, tower, and projectile guide.
2. Select the projectile arm and L_1.
3. Set the tower height to obtain θ.
4. Install the counterweight.
5. Assemble the projectile and its bridle.
6. Install the projectile/bridle on the trebuchet arm to obtain η.

7. Adjust the projectile guide position.
8. Arm the trebuchet. Hold it in its armed position by pinching the projectile to the guide.
9. Warning and countdown.
10. Measure and record the response.

1.5 General Procedure for Experimentation

3. Write a Detailed Problem Statement

- Select the response(s) for study
- Identify the variables for study
  - Variables for active experimentation
  - Variables to be held fixed
  - Variables that cannot be controlled but can be recorded (covariates)
- Identify possible interactions between variables
- Identify critical assumptions
- Identify knowledge gaps
- Use all available historical data and expert opinions

1.5.1 Trebuchet DOE: Problem Statement

Perform a resolution IV screening experiment of the following trebuchet variables: L_1, L_2, L_3, W_1, W_2, θ, η. Hold all other variables fixed, where possible. Assume that the counterweight arm length and friction are negligible. The response should be the final resting position of the projectile. Perform the experiment in an environment that will prevent interference with the projectile.

1.6 General Procedure for Experimentation

4. Preliminary Experimentation

- Perform a gage error study
- Estimate the repeatability
- Demonstrate reproducibility
- Demonstrate that the process is in control
- Determine levels for input variables

  - Quantitative or qualitative
  - Fixed or random
  - Too narrow and you won't see an effect
  - Too wide and you may lose runs or get curvature
- Use no more than 20% of your resources
- Refine the experimental procedure
- Confirm that all equipment is operating correctly and has been maintained

1.6.1 Trebuchet DOE: Preliminary Experiments

A preliminary experiment (4 February) uncovered several problems with the original trebuchet design and the experimental process. As a result of those observations, the following actions were taken:

- Trebuchet base redesign.
- Redesigned process of obtaining θ by controlling the pivot point height.
- Redesigned the projectile bridle design and to simplify assembly and improve accuracy.
- Need a larger room.
- Amount of noise in the response is large so launch distance measurement scale resolution can be 1/4 foot.

1.7 General Procedure for Experimentation

5. Select an Experiment Design

- The choice of design type ranges from screening designs to response surface designs:
  - Highly fractionated factorial design for main effects only
  - Factorial design for main effects and interactions
  - Response surface design for main effects, interactions, and quadratic terms.
- Match the choice of design type to the expected behavior of the response.
- Take advantage of your technical knowledge of the system when considering which design to choose.
- The simplest designs are the ones that are used most frequently.
- Use no more than 70% of your resources

1.7.1 Trebuchet DOE: Experiment Design

Used a 2_{IV}^{7-3} design with two replicates. Center cells were not possible because one of the design variables (η) does not have an intermediate level available. Chose two replicates of the 2_{IV}^{7-3} instead of one replicate of a 2_{IV}^{7-2} because it reduced the number of arms required to four and the number of projectile bridles to six.

1.7.2 Trebuchet DOE: Experiment Design

Variables matrix:

Variable	-1	$+1$	Units
L_1	2	4	$inch$
L_2/L_1	3	5	NA
W_1	2	3	#6 weights
W_2	1	3	$1/4in$ SS washers
L_3/L_2	0.5	1	NA
θ	30	60	$degrees$
η	0	45	$degrees$

1.8 General Procedure for Experimentation

6. Sample Size, Blocking, and Randomization

- Determine the number of replicates with a sample size calculation
- Build large experiments in blocks, e.g. block on replicates
- Randomization is difficult, expensive, and painful but you MUST randomize
- Validate your randomization plan
- Failure to randomize may lead to incorrect conclusions
- Failure to randomize leaves your conclusions open to challenge
- Failure to randomize may force you to repeat the experiment

1.8.1 Trebuchet DOE: Sample Size, Randomization, and Blocking

Due to time and hardware limitations and the screening nature of the experiment only two replicates of the 2_{IV}^{7-3} design were built. The two replicates were built in separate blocks and runs were randomized within blocks.

1.9 General Procedure for Experimentation

7. Conduct the Experiment

- Make sure all critical personnel are present
- Record all of the data
- Record the run order
- Note any special occurrences
- Fix and note any problems that are detected as soon as possible. It may be necessary to start the experiment over again.

1.9.1 Trebuchet DOE: Experiment Execution

- Difficulties deciding how to interpret the launch distance:
  - Projectile sometimes landed on table.
  - Some serious bounces.
  - Some serious slides.
  - Two significant rolls.
  - Need to redesign to obtain the first point of impact by either 1) launching over sand? or 2) using a soft projectile
- Counterweight interference with projectile guide
- Projectile guide too high and too short
- Counterweights too tall for some geometries
- Consider $\theta \geq 45°$.

1.10 General Procedure for Experimentation

8. Analyze the Data

- Graph the data (DTDP!)
- Run the ANOVA or regression analysis
- Check assumptions:
  - Orthogonality
  - Equality of variances
  - Normality of residuals
  - Independence

  - Goodness of fit

- Refine the model using Occam's Razor: The simplest model that explains the data is probably the best model.
- Determine the model standard error and R-squared

1.11 General Procedure for Experimentation

9. Interpret the Results

- Select the best variable levels
- Predict the response
- Don't extrapolate outside the range of experimentation
- Plan a follow-up experiment to resolve ambiguities

1.12 General Procedure for Experimentation

10. Perform a Confirmation Experiment

- Use a confirmation experiment to validate the model
- Don't prematurely report your results until the confirmation experiment results are in
- Use the remaining 10% of your resources

1.13 General Procedure for Experimentation

11. Document the Results

- Keep all of the original records and notes
- Write the formal report
- Know your audience

1.14 Components of a Formal Report

- Administrative information: by, for, date, title, contract number
- Findings or abstract: executive summary describing the experiment and its results, not to be more than 1/2 page long.
- Introduction or background: describes the system being studied, relevant conditions, and the purpose of the report.

- Experiment design: describes the experiment design, sample size, randomization, and blocking plans.
- Experimental data: describes and presents the data with any special causes.
- Analysis: presents the complete statistical analysis and interpretation including tables and figures.
- Recommendations: describes identified improvements to the design of the experiment or the process being studied. If necessary, will call for a confirmation experiment.