Design of Experiments With MINITAB: Classroom Exercises and Labs (Rev. 20211230)

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Contents

Preface

This document describes demonstrations that can be used in the classroom or that are suitable as lab projects in a DOE class. These demonstrations are designed to be simple to perform and require minimal time and equipment. Many of the demonstrations make use of experiments with paper helicopters, dice, or a toy catapult. While many students are initially skeptical about the value of these experiments, they soon discover that these experiments are much more difficult than they originally realized and provide tremendous opportunites to develop and hone DOE skills. The value of these demonstrations is proved by their almost universal use in training classes.

To simplify experiments with paper helicopters, three helicopter templates on graph paper are provided in the files *helicopter1.doc*, *helicopter2.doc*, and *helicopter3.doc* and samples are shown in Figure 1. (The stick on the last helicopter is a 3 inch bamboo skewer.) These templates can be printed or copied onto papers of various weights and colors. Then students can cut them out and fly them from fixed or various altitudes to determine responses like flight time, stability, and horizontal deviation. Flight time measurements should be taken with a digital stopwatch capable of resolving tenths or hundreths of seconds. Horizontal deviation can be measured with a tape measure or yardstick.

Experiments with dice can consider several potential responses: the mean, the standard deviation, the range, the number of occurrences of a specified condition (e.g. the number of threes), or the sum of the die faces. One of the easiest responses

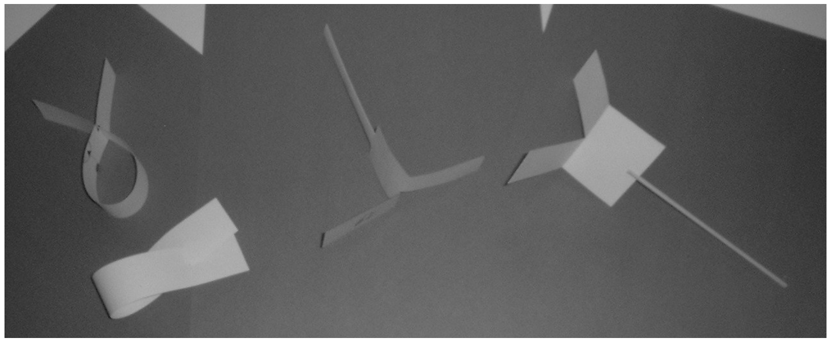

FIGURE 1. Samples of helicopters made from templates *helicopter1.doc*, *helicopter2.doc*, and *helicopter3.doc*, respectively.

for students to understand is the sum of the faces of six rolled dice. It's also easy to create a series of increasingly complex but related experiments with dice.

Several of the dice experiments make reference to "magic" dice. These dice are modified standard dice where some of the faces of dice are changed from ones to fives, twos to fives, threes to fives, and fours to fives. Some standard and magic dice are shown in Figure 2. New dimples in die faces can be made in several ways. The most elaborate way is to use a ball end mill in a drill press or milling machine to add dimples to selected die faces and paint the new dimples with an appropriate color nail polish. Alternatively, new dimples can just be added using dots of nail polish; however, the more the modified dice look like standard dice, the harder it is to detect the modifications. This makes it easier to surprise students with experimental data - everyone assumes that they know how dice work - but the magic dice can often be slipped in without detection, especially if they are mixed with a few standard dice to deliver the few ones, twos, threes, and fours necessary to alleviate any early suspicions that the dice might be weighted. Special dice with different face designs or even blank faces for custom projects can be purchased from an educational supply store.

Many DOE classes use toy catapults. These catapults provide a wonderful vehicle for teaching DOE. They have many easy to understand and adjust design variables and a simple to understand and measure response - the total flight distance. The flight distance can be measured with a tape measure or by counting tiles if the experiment is performed on a tiled floor. There are also many unanticipated process variables that affect the response that students have to discover by experience and careful observation.

FIGURE 2. Standard and magic dice.

I built the catapults that I use in my classes in my garage using materials available from any home improvement store and common hand tools. My standard catapult design is shown in Figure 3. Each catapult requires the following materials:

- Base (1) - 0.75 x 4.5 x 20 inch pine board
- Start peg socket (1) - 1.5 x 3.5 x 4.5 inch pine with 0.75 inch hole in center
- Hook tower (1) - 1.5 x 3 x 12 inch pine
- Arm (1) - 0.625 x 1.75 x 22 inch pine
- Pegs (2 of each size) - 0.75 in diameter by 3, 5, 7, and 9 inches long
- Hinge (1) - 1.5 inch wide by 4 inch long steel hinge
- Hooks (6) - 3 in long open-eyed steel hooks
- Projectile cups (3) - 1 inch PVC pipe caps
- Screws - Various sizes and lengths to: 1) attach the tower to the base, 2) attach the start peg socket to the base, 3) attach the hinge to the arm and base, and 4) attach the PVC caps to the arm

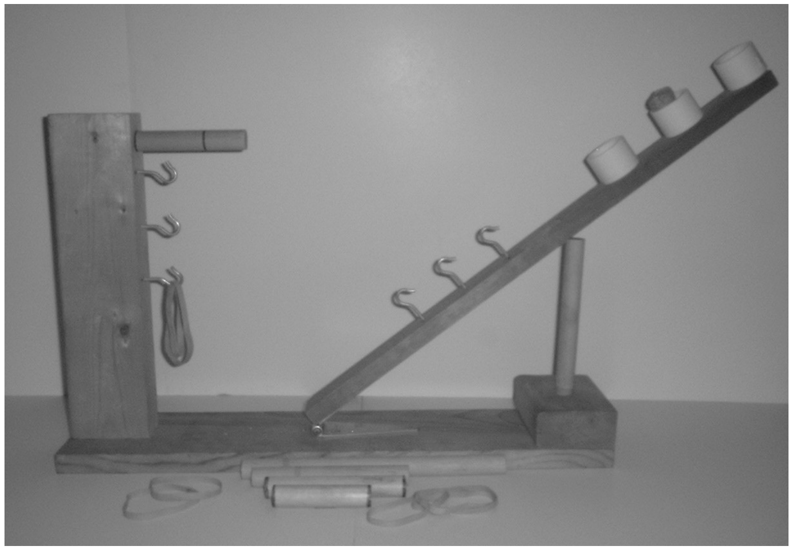

FIGURE 3. Catapult.

- Rubber bands - Standard office rubber bands 0.25 inch wide by 3.5 inches long
- Tin foil - Wadded up to create the projectile

Although one individual or team can experiment with any of these devices, perhaps the most informative approach to learning DOE is to challenge several small teams to compete with each other to meet a specified design goal. Each team should be given a limited amount of time and resources and required to present their strategy, DOE program, and results to the whole class so that everyone see the benefits and pitfalls of various approaches. There is much more value in using the competing teams approach than by having an individual or single team perform these exercises alone.

Every DOE class has some skeptics who doubt the value of DOE methods and the usefulness of the exercises with helicopters, dice, and catapults; however, many of these people are easily convinced of the value of DOE methods after they complete these exercises and sometimes the most staunch defenders of the old methods become the most vocal converts to DOE. The final key to cementing the faith of those newly trained in DOE methods is to have them identify and execute a successful DOE program to study an important process that has eluded earlier attempts to analyze or refine it.

1

Graphical Presentation of Data

1. Cut-out and fly the paper helicopter in *helicopter1.doc* from a constant altitude and measure the flight time. Use a digital stopwatch to determine the flight time to tenths or hundredths of seconds. Record the flight times for twelve flights and then create a dotplot, stem-and-leaf plot, histogram, and boxplot of the data.

2. Roll six dice twenty times and record the sum of the die faces for each roll. Create a dotplot, stem-and-leaf plot, histogram, and boxplot of the data.

3. Configure a toy catapult to its nominal settings. Launch the projectile from the catapult twelve times and record the distance to the final resting position of the projectile. Create a dotplot, stem-and-leaf plot, histogram, and boxplot of the data.

4. Roll six dice including from zero to six magic dice five times under each condition and plot the sum of the die faces as a function of the number of magic dice.

5. Print the paper helicopter design from *helicopter2.doc* on both 20 and 24 pound paper. Mark the helicopters carefully or use different paper colors to avoid confusing them. Cut out a total of eight paper helicopters, four from each paper weight, where the helicopters have: 1) nominal blade length and nominal blade width, 2) nominal blade length and narrow blade width, 3) short blade length

and nominal blade width, and 4) short blade length and narrow blade width. Fly all eight helicopters several times from the same altitude and measure their flight times. Prepare an appropriate graphical presentation of the data and describe the effects of paper weight, blade length, and blade width on flight time.

2

Descriptive Statistics

1. Calculate the sample mean and standard deviation of the helicopter flight time data from Problem 1.1. Use the sample range to estimate the population standard deviation and compare it to the sample standard deviation.

2. Calculate the sample mean and standard deviation of the dice experiment from Problem 1.2. Use the sample range to estimate the population standard deviation and compare it to the sample standard deviation.

3. Calculate the sample mean and standard deviation of the distance data from Problem 1.3. Use the sample range to estimate the population standard deviation and compare it to the sample standard deviation.

3
Inferential Statistics

1. The claim is made that the paper helicopter in *helicopter1.doc*, when copied onto 20 pound paper and flown from eight feet, should have a flight time of $\mu = 2.2$ seconds. Test this claim by making and flying one helicopter eight times. Use a normal plot to test the normality assumption.

2. Use the standard deviation of the helicopter flight time from Problem 3.1 to determine the sample size required to resolve a deviation from $\mu = 2.2$ of $\delta = 0.1$ second with 90% power. If this sample size is not practical, revise the problem, fly the helicopter the required number of times, and perform the test. Use a normal plot to test the normality assumption.

3. Determine the number of pennies required to stretch a standard 0.25 inch by 3.5 inch rubber band to a length of 5.0 inches by adding one penny at a time to the rubber band. Repeat the evaluation eight times, present the data graphically, and calculate the mean and standard deviation. Construct the 95% confidence intervals for the population mean and standard deviation.

4. Roll six standard dice several times to estimate the standard deviation of the sum of the die faces. Use this estimate to determine the sample size required to detect a bias of $\delta = 2$ of the mean sum from the expected mean of $\mu = 21$. Roll the dice the specified number of times and perform the test of $H_0 : \mu = 21$ versus $H_A : \mu \neq 21$. Use a normal plot to test the normality assumption.

5. Use the estimate of the standard deviation of the sum of the die faces response from Problem 3.4 to determine the sample size required to detect a bias of $\delta = 2$ between the mean sums of two different sets of six dice. Perform the experiment and test the hypotheses $H_0 : \mu_1 = \mu_2$ versus $H_A : \mu_1 \neq \mu_2$.

6. Test the normality and homoscedasticity assumptions of the two-sample data from Problem 3.5. Which form of the two-sample t test should be used in that problem to have the most sensitivity to small biases? Is this method valid and why?

7. Obtain ten small paper clips and ten large ones. Bend each clip open at its center until its two halves are 180 degrees apart. Holding the ends of the clip in two hands, bend the clip from 180 degrees to 90 degrees and back. Count the number of bending cycles required to break the clip. Repeat the process for all of the clips and test the data for a difference in the mean number of cycles to failure. By two-sample t test.

8. Instead of having one person break all of the paper clips in Problem 3.7, have each person in the class break one small and one large paperclip. Is there a difference in the mean number of cycles to failure? By paired-sample t test. Revisit at two-way ANOVA in Chapter 6.

9. Comment on the following paper clip experiment: ten large and ten small paper clips are assigned randomly to twenty students who recieve one paper clip each.

4

DOE Language and Concepts

1. For a paper helicopter:

   (a) Brainstorm a list of potential variables that could affect the flight of the helicopter and a list of potential responses. Present your results in the form of a modified cause and effect (or IPO) diagram.

   (b) Create a flow chart for the process of flying helicopters to determine their flight time.

2. Brainstorm a list of potential variables that could affect the configuration and operation of a catapult and a list of potential responses. Present your results in the form of a modified cause and effect (or IPO) diagram.

3. Use the 11-step DOE process to perform an experiment to test for a bias between two sets of six dice. Use the sum of the six die faces as the response. Justify your choice of sample size.

4. Use the 11-step DOE process to study the flight time difference between two sets of helicopters that have different paper weights. Justify your choice of sample size.

5. Use the 11-step DOE process to study the flight time difference between one helicopter configured for clockwise and counterclockwise rotation. Justify your choice of sample size.

6. Use the 11-step DOE process to study the launch distance difference between two catapult configurations. Justify your choice of sample size.

7. How would you design an experiment to determine if your underwear and t-shirts change states - from right-side out to inside out - when they are washed and dried? What type of variable is the response?

8. For the process of breaking paper clips:

    (a) Create the input/process/output (IPO) diagram for the process of breaking paper clips.

    (b) Create a flow chart to document the process.

9. Use your helicopter-flying or paper clip-breaking flow chart to train a novice in that activity. Report any new variables that you discover and any changes to the original document that you find necessary to clarify the process.

5

Experiments for One-Way Classifications

After completing each experiment, create a list of mistakes that you made or almost made and appropriate corrective or preventative actions.

1. Give a set of six standard dice to each of five persons or teams. Have them roll their dice ten times while recording the sum of the die faces. Analyze the data using boxplots and by one-way ANOVA.

2. Repeat the experiment from Problem 5.1, but replace one of the sets of six standard dice with another set including three standard and three magic dice. If you were unaware of the difference between the sets of dice, how would you have to interpret the results of the ANOVA and post-ANOVA multiple comparisons? What is the flaw in this experiment design?

3. Prepare four lists of ten items each. One list should be of types of fruit, one should be of common names for dogs, one should be of common models of cars, and the last should be of large cities. Give test subjects two minutes to study one of the lists, then wait two minutes before having them recall as many of the items as they can. Do test subjects have better recall for some types of lists than others?

4. In a team of four or more people, have one person measure the flight time of a paper helicopter as a function of the other team members. Use one-way

ANOVA and post-ANOVA multiple comparisons to determine if there are flight time differences between the members.

5. Reconsider the paper clip breaking experiment from Problem 3.7. What experiment design did you use and what analysis method? Were the design and analysis method appropriate for the goal of the experiment - to determine if there was a difference in the toughness between small and large paper clips? If your original design or analysis method were inappropriate, design and execute a new better experiment.

6

Experiments for Multi-Way Classifications

After completing each experiment, create a list of mistakes that you made or almost made and appropriate corrective or preventative actions.

1. Redesign the one-way classification experiments from Problems 5.1 and 5.2 but use a two-way classification design to separate the effects of dice and people/teams. Perform the experiments and contrast your interpretation of the results to those from the original experiments.

2. Perform an experiment to study paper helicopter flight time as a function of design, person dropping the helicopter, and person timing the flight. Use the nominal configuration of *helicopter1.doc*, *helicopter2.doc*, and *helicopter3.doc* with a 2 inch bamboo skewer for the third design for the three levels of the design variable and use at least two levels of dropper and timer.

3. Redesign the experiment from Problem 5.3 as a multi-way classification design and execute the new experiment. How should the order of the lists be managed?

4. Obtain paper clips of at least two different sizes and select three or more people to break them. Design and execute a two-way classification experiment to determine if there are location differences between types of paper clips and biases between operators.

7

Advanced ANOVA Topics

After completing each experiment, create a list of mistakes that you made or almost made and give appropriate corrective or preventative actions.

1. Build a Latin Square experiment to study the sum of the die faces response for six dice using the following three variables: 1) three sets of dice (all standard, all standard, 3 standard plus 3 magic), 2) three rollers, and 3) three drop heights (low, moderate, and high). Analyze the experimental data. Do you obtain the expected results?

2. Build a balanced incomplete block design to study the sum of the die faces response for six dice using five people or teams to roll dice and five sets of dice. One of the five sets of dice may be magic dice. Each person or team should roll four of the five sets of dice. How does the model that you fit to these data compare to the model fitted to the balanced full factorial design?

3. Select six standard dice and mark each with a unique number from 1 to 6. Perform a gage error study with two or more operators using a set of calipers to measure the thickness of the dice across the 3 / 4 die faces. Be careful to orient the caliper jaws to cover the three dots on the 3 face of the die to guarantee that everyone is measuring at the same place in case the die faces are not parallel. Assume that the tolerance width is 0.030 inches.

4. Repeat the GR&R study from Problem 7.3 but this time orient the calipers on each die so that the measuring surfaces cover the three dimples of the three face on each die. Analyze the data and compare your results to those from the original study. Based on your observations, make some recommendations about part geometry and operator instructions for gage error studies.

5. Perform a nested GR&R study by giving two or more people their own sets of six dice. Each person should measure each die at least two times and each measurement should be taken over the three dimples of each die.

6. Have one person drop a paper helicopter several times and have several people measure the flight time simultaneously. Analyze the data using two-way ANOVA and estimate the repeatability and reproducibility.

8
Linear Regression

After completing each experiment, create a list of mistakes that you made or almost made and appropriate corrective or preventative actions.

1. Roll six dice including from zero to six magic dice five times under each condition and model the sum of the die faces as a function of the number of magic dice. (Use the data from Problem 1.4 if you can find them.)

2. Perform an experiment to determine the spring constant of a rubber band. (The spring constant is the slope of the force versus displacement curve in its linear region.) Use coins as weights and use paper clips as hooks. A picture of the experiment apparatus is shown in Figure 8.1.

3. The stretch observed in the rubber band in Problem 8.2 is time dependent - the rubber band continues to stretch well after the load is applied. Use the apparatus from Problem 8.2 to determine the relative change in rubber band length from two seconds versus 30 seconds post loading as a function of the applied weight. Is the relative change constant with respect to increasing loads or does it change?

4. Obtain a piece of modeling clay or Play Doh. Place the clay on a piece of graph paper, put a piece of wax paper over the clay, and then place a flat board or book on the wax paper. Have a series of students (who are not concious about their weights) stand on the board until the clay stops flowing, then measure the

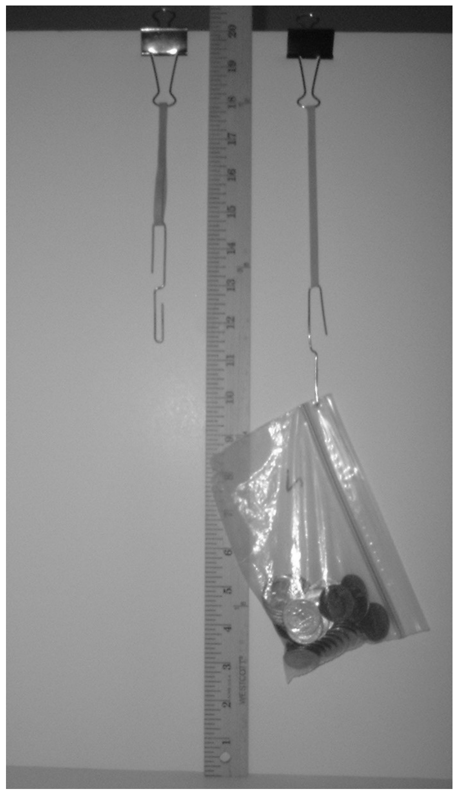

FIGURE 8.1. Rubber band loading and elongation.

area of the clay that supported their weight. Fit a model for area as a function of applied weight.

5. Drop a ping-pong ball from a fixed height onto a hard surface covered with varying thicknesses of stacked paper and measure the ball's rebound height. (This can be done quite accurately using a video camera.) Construct a model for rebound height as a function of paper thickness.

6. Obtain a magnet and a suitable heavy steel object. Suspend the magnet from the rubber band using the apparatus of Problem 8.2. Use this arrangement to study the rubber band's stretch as a function of the distance between the magnet and the steel.

7. Obtain a large diameter PVC pipe with a glued-on waterproof cap on one end. Drill a series of small (0.25 inch) evenly spaced holes along the length of the pipe from one end to the other. Put electrical tape over each hole. Stand the pipe on end and fill it with water. Uncover each hole and measure the horizontal range of the resulting stream of water. Fit a model for the horizontal range as a function of hole altitude.

8. Modify the PVC pipe from Problem 8.7 by drilling two new sets of holes of 0.125 inch and 0.375 inch diameter at the same altitudes as the original holes. Fit a suitable model for the horizontal range as a function of altitude and hole diameter.

9. Have three or more people each roll from one to six standard dice three times each (i.e. 18 rolls per person) and record the sum of the die faces for each roll.

   (a) Fit a model to the sum response as a function of person as a qualitative variable and number of dice as a quantitative variable. Include terms in the model for the person by number of dice interaction and a quadratic term for the number of dice.
   (b) Refine the model.

10. Drop a single paper helicopter from various altitudes and record the flight time. Build a linear regression model for flight time as a function of altitude.

11. Repeat Problem 8.10 but have different people drop the helicopter from each altitude. Analyze the flight time response as a function of altitude (as a quantitative variable) and person (as a qualitative variable). Include the altitude by person interaction term and, if necessary, a term for blocks in the model.

9

Two-Level Factorial Experiments

After completing each experiment, create a list of mistakes that you made or almost made and appropriate corrective or preventative actions.

1. Build a 2^4 experiment to study flight time as a function of blade length, blade width, slot position (from the blade ends), and slot depth of the helicopter design from *helicopter1.doc*.

2. Use the following steps to investigate the flight time response of paper helicopters.

   (a) Build a nominal paper helicopter and fly it 8-10 times to estimate the standard deviation of its flight times. Use that estimate to determine how many replicates of a 2^3 experiment design are required to resolve a difference of 0.25 seconds in flight time between the low and high states of study variables with 90% power.

   (b) Build a 2^3 experiment to study paper helicopter flight time as a function of three variables: blade length, blade width, and paper clip (with versus without). Use the number of replicates determined in Part a and block the experiment on replicates. Analyze the experimental data and refine the model.

   (c) Use the model from Part b to determine the geometry of the paper helicopter that maximizes the flight time. Use the model to predict the flight time for that geometry, then build and fly that helicopter several more times. Use the one-sample t test to test the new data for compliance to the prediction.

3. Build a 2^5 experiment to study the launch distance of the catapult as a function of start peg length, stop peg length, tower hook position, arm hook position, and cup. Hold the catapult arm back for two seconds before releasing it and step on the catapult base to prevent it from recoiling.

4. Use the apparatus from Problem 8.8 to study the horizontal range as a function of hole diameter, water temperature, and liquid soap (with and without) using a 2^3 design. Use a single altitude that gives near maximum horizontal range.

5. Build a two-level full factorial experiment to study the rebound height of golf balls as a function of golf ball (two types), altitude, number of sheets of paper under the dropped ball, and operator. Build models for both the absolute and relative rebound height.

6. Tape several coins together to make a hockey puck and use a flexible ruler pressed to the edge of a table top to slap it across the table. The experimental response is the distance that the puck travels. Brainstorm a list of variables the might affect the distance response and perform an appropriate 2^k experiment to study the system.

7. Download BalloonCarBuilder.exe from http://pbskids.org/zoom/games/. Install and run the program. Use an appropriate experiment design to evaluate the design variables for virtual balloon cars. Use your model to predict the car geometry that maximizes the speed and distance that the car travels. What happens when you replicate runs from your experiment design?

10

Fractional Factorial Experiments

After completing each experiment, create a list of mistakes that you made or almost made and appropriate corrective or preventative actions.

1. Perform the following paper helicopter screening experiment:

   (a) Design an experiment using a highly fractionated two-level factorial or Plackett-Burman design to study paper helicopter flight time as a function of: blade length (long/short), blade width (narrow/wide), paper clip(without/with), blade fold direction(CW/CCW), paper weight (heavy/light), operator(dropper/timer (swap roles for two levels)), and pinch position (middle of body/blade tips). The experiment design should be at least Resolution IV.

   (b) Determine the number of experimental runs required to detect a 0.4s flight time effect with 90% power using the standard error from a previous experiment. Determine how many replicates of the experiment design from part a) are required to reach this total number of runs.

   (c) Build and fly the paper helicopter experiment. Don't do any more than about 32 runs if part b) calls for more than that.

   (d) Analyze the data, refine the model, and try to find a defensible interpretation for the regression coefficients that you obtain. List the most significant variables by the magnitudes of their effects. Recommend a follow-up experiment.

2. Perform a 2_V^{5-1} plus centers design to study five of the most important paper helicopter design variables identified in Problem 10.1. Keep the helicopters that you build and the experimental data and analysis because you may use them again in a follow-up experiment.

3. Perform a screening experiment to study at least seven variables that could affect catapult launch distance. Rank order the variables in terms of the magnitude of their effects.

4. Build a 2_V^{5-1} plus centers experiment design to study the five most important catapult variables. Is there evidence of curvature in the response with respect to one or more of the design variables?

5. Select and analyze one half-fraction of the original 2^4 experiment from Problem 9.1 and compare your results to the original full factorial design. Does the half fractional factorial experiment resolve all of the important effects?

11
Response Surface Experiments

After completing each experiment, create a list of mistakes that you made or almost made and appropriate corrective or preventative actions.

1. Perform an experiment to study the maximum deflection of a centrally loaded rectangular beam with ends that are free to rotate. At a minimum, the experiment should consider the following design variables: beam height (h), beam width (w), beam span (L), and applied force or load (F). Use balsa or basswood sticks available at any hobby shop or craft store as beams and coins as weights. Figure 11.1 shows the hardware configured for an experimental run. The beam is loaded with quarters suspended from a binder clip and the deflection is measured with a vertical steel rule taped to the yard stick at the mid-span of the beam. Use an appropriate response surface experiment design and fit an empirical model to the data.

2. The theoretical equation for the maximum deflection Δ of a centrally-loaded simply-supported rectangular beam is given by:

$$\Delta = \frac{1}{4E}\frac{FL^3}{wh^3} \tag{11.1}$$

   where E is the elastic modulus of the beam material. Use a model of this form to fit the data from Problem 11.1. (Hint: Apply a logarithmic transform to Equation 11.1.) How well do the regression coefficients match the theoretical

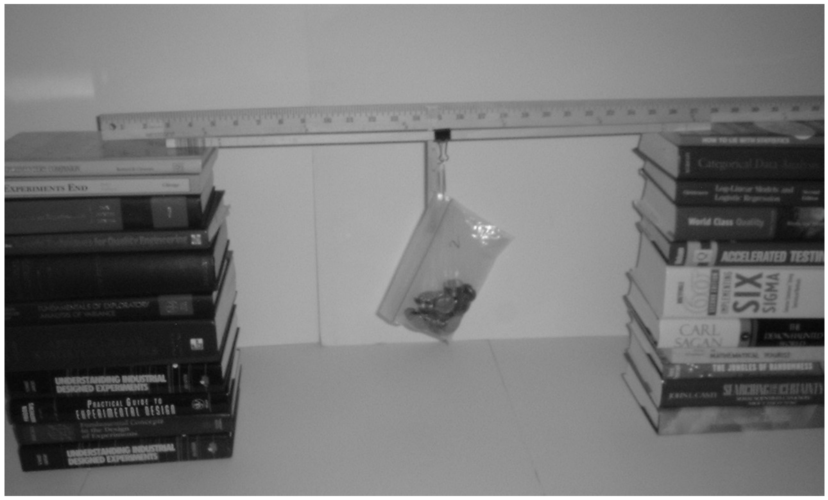

FIGURE 11.1. Measuring the deflection of a centrally loaded rectangular beam.

model? What conditions in the experiment might cause any observed discrepancies? (Hint: Equation 11.1 only accounts for relatively small deflections.)

3. Design and execute a new experiment to study beam deflection. In this experiment, the load F should be split into two equal halves with the halves placed symmetrically about the beam center displaced a variable distance $\pm c$ from the beam center. Include the original four design variables in the new experiment to determine if this new variable changes their influence on the deflection.

4. Perform a response surface experiment to study the flight time and radial displacement of one of the paper helicopter designs. Use your experience from earlier experiments to limit the number of variables considered, but include altitude as one of the experimental variables.

    (a) For a specified altitude, what are the conditions that maximize the flight time?

    (b) For a specified altitude, what are the conditions that minimize the radial displacement?

    (c) Pick a target flight time within the observed range of flight times and build a new helicopter that will deliver that flight time. Calculate the confidence and prediction intervals for the target flight time based on your model and then check to see if your new helicopter design is consistent with these intervals.

5. Perform a response surface experiment to study the key design and process variables of the catapult. Use your experience from earlier experiments to limit the number of variables considered, but include the number of rubber bands as a design variable.

   (a) What conditions maximize the launch distance?

   (b) Pick a target flight distance and then configure the catapult to deliver that distance. Calculate the confidence and prediction intervals for the target launch distance based on your model and then check to see if your new helicopter design is consistent with these intervals.

6. The viscosity of water, which determines its frictional properties, is dependent on temperature. Repeat the experiment from Problem 8.8 using a Box-Behnken three variable design with water temperature as the third variable. Is there a measureable effect of viscosity on horizontal range?

7. A 2_V^{5-1} plus centers design was built to study paper helicopter design variables in Problem 10.2. Build the extra runs required for a $CC\left(2_V^{5-1}\right)$ experiment and add them to the original data. Be sure to analyze the experiment as two blocks - the first block consisting of the 2_V^{5-1} plus centers design and the second block consisting of the star and additional center points.

8. Clamp or support a wide brim funnel with its tip down and its rim horizontal. Position a 12 inch long by 3/4 inch ID piece of PVC pipe so that one end of the pipe is over the lip of the funnel and the other end is slightly elevated. Roll a marble or ball bearing down the pipe into the funnel and measure the amount of time it takes for the marble to drop out of the funnel. Adjust the angle of the pipe with respect to the funnel and the elevation change over the length of the tubing to maximize the marble's dwell time in the funnel.

9. Repeat Problem 11.8 but consider the effect of different marble or bearing diameters on the dwell time.

10. Build a response surface experiment to study the rebound height of golf balls as a function of the drop altitude and the number of sheets of paper placed under the dropped ball.

11. Build an experiment to study the peal strength response of sticky notes stuck to a desk top as a function of pull force and angle. Use a spring scale or a string and weight system to apply the pulling force to the sticky note.

12. Drop ball bearings or marbles into a tray of sand and measure the diameter of the impact crater. Build an experiment to analyze impact crater diameter as a function of ball bearing size, drop altitude, sand type (e.g. course vs. fine), sand depth, and any other design or process variables that you can easily integrate into the experiment.